Tips for task 3

# Exercise 1

Important: Suppose that the samples are independent and identically distributed. Our likelihood function has then the following form:

Do not forget that is the precision matrix. As usual, because of the exponential function and the positivity of the function it is easier to derive

1. Take the derivative of with respect to the appropriate variable
2. Solve
3. Check if solution is indeed extreme value

Again, do these steps for**,**  separately i.e. work with partial derivatives.

Let us do it for . In contrast to the last exercise, is now a vector and not a scalar. Because of each component obeys symmetry properties concerning the other , we can just take the partial derivative of one component and stack the final solution into a vector.

It is necessary to conduct some auxiliary calculations:

|  |
| --- |
| Which leads to |

Where denotes the dot product of the th row of with . Also, do not forget that as is symmetric!

This allows us to determine the whole gradient:

.

Since has inverse we just multiply the equation from the left side with **:**

Now do the same for the covariance matrix!

# Exercise 3

First, calculate the product:

Now calculate the exponent and solve for . The procedure is called “completing the square “(google it). Do not forget: We are only looking for proportionality, so you can drop all terms that do not depend on

Now complete the square of the term in the numerator. If you look closely, you can already recognize the wanted expectation and variance.

# Exercise 4

What a wonder! Maximum likelihood! I wonder what we have to do now? Of course the same as always! This time we do not have to use the logarithm, as now it would actually make our job harder!

Hint: Instead of solving directly, again solve the partial derivatives and combine final solution into a vector (or matrix) if possible.

Now solve for :

It appears that is dependent on the other ! Are we now stuck? Look at the mean calculation in exercise one, where we converted our equation into a matrix. Is it possible here?

To answer that what do you think about the following decomposition:

Where

Is the design matrix.

If this decomposition is correct, this would lead to:

Can we solve this for **?** Hint: It might be possible to use the Penrose Pseudoinverse if a matrix is originally not invertible.